SHORTRUN RESPONSE OF HOUSING MARKETS TO DEMAND SHIFTS

C. PETER RYDELL

R-2453-HUD

SEPTEMBER 1979

HOUSING ASSISTANCE SUPPLY EXPERIMENT

Sponsored by

The Office of Policy Development and Research

U.S. Department of Housing and Urban Development

Rand

SANTA MONICA, CA. 90406

RCN 9016

R-2453-HUD SHORTRUN RESPONSE OF HOUSING MARKETS TO DEMAND SHIFTS. C. Peter Rydell. September 1979.

Presents a theory of shortrun market adjustments to exogenous demand shifts that is consistent with evidence from the Housing Assistance Supply Experiment (HASE). It is argued that (1) a 1.0 percent shift in rental demand leads to a rent change of only 0.26 percent, whereas capital value can change as much as 5.0 percent; and (2) landlords derive capital value in a tightening market primarily from decreased vacancy loss (because of monopolistic competition among themselves) rather than from increased nominal rent. Using HASE data, the theory predicts that a housing allowance program would cause shortrun rent increases of 0.6 to 1.0 percent and capital value increases of 1.6 to 6.5 percent, depending on the size and duration of allowance-induced demand shifts. 43 pp. (CC)

The research reported here was performed pursuant to Contract No. H-1789 with the Office of Policy Development and Research, U.S. Department of Housing and Urban Development. Statements and conclusions in this report are those of Rand's research staff and do not necessarily reflect the views of the sponsoring agency.

Library of Congress Cataloging in Publication Data

Rydell, C Peter.
Shortrun response of housing markets to demand shifts.

Rand Corporation ; R-2453-HUD)
--Mathematical models. 2. Supply
.thematical models. 3. Rent--
iodels. I. United States.
ing and Urban Development.
III. Series: Rand Corporation.
R-2453-HUD.
53 [HD7287] 081s 79-23021
171-X [381'.45'6908]

Published by The Rand Corporation

SHORTRUN RESPONSE OF HOUSING MARKETS TO DEMAND SHIFTS

C. PETER RYDELL

R-2453-HUD

SEPTEMBER 1979

HOUSING ASSISTANCE SUPPLY EXPERIMENT

Sponsored by

The Office of Policy Development and Research
U.S. Department of Housing and Urban Development

Rand
SANTA MONICA, CA. 90406

PREFACE

This report was prepared for "The Housing Choices of Low-Income Families: Analysis from the Experimental Housing Allowance Program," a conference sponsored by the Office of Policy Development and Research, U.S. Department of Housing and Urban Development (HUD). The conference was held in Washington, D.C., on 8-9 March 1979.

The report draws on research conducted by Rand as part of the HUD-sponsored Housing Assistance Supply Experiment, which in turn is part of HUD's Experimental Housing Allowance Program. Comments by the following people on various drafts of this work led to a considerably strengthened report: C. Lance Barnett and Ira S. Lowry at The Rand Corporation; John Cogan at Stanford University; Bryan Ellickson at the University of California, Los Angeles; Edwin Mills at Princeton University; and Larry Ozanne at the Urban Institute. Jan Newman typed the various drafts. Robin Boynton was the production typist. Charlotte Cox edited the report and supervised its production.

The report was prepared under HUD contract H-1789.

SUMMARY

In the shortrun, although the supply of housing services is fixed, the occupied supply can vary. That variation explains the small effect of demand shifts on the rent paid for a given amount of housing service. When the demand curve for rental housing shifts, the vacancy rate absorbs much of the effect. Only a little movement along the demand curve, and consequently only a small rent change, is required to bring occupied supply and realized demand into shortrun equilibrium. The shortrun insensitivity of rent to demand shifts contrasts with the sensitivity of capital value. It is estimated that a 1.0 percent shift in rental demand causes only a 0.26 percent change in rent, but that it can cause a 5.0 percent change in capital value.

Under a monopolistic competition theory of housing market behavior, shortrun equilibrium is shown to depend on two price elasticities of demand: the price elasticity of aggregate demand, a familiar concept; and the additional price elasticity of demand for an individual landlord's housing services, a new concept. The theory shows not only that market rent is less than it would be under a monopoly and greater than it would be under perfect competition, but also that the effect of a demand shift on rent is greater than it would be under a monopoly and less than it would be under perfect competition.

Applying the theory to the Housing Assistance Supply Experiment, and using experimental data to estimate the parameters, yields predictions that introducing a housing allowance program would cause shortrun rent increases of only 0.6 to 1.0 percent, and capital value increases of 1.6 to 6.5 percent. Variation in the size and duration of allowance-induced demand shifts in portions of the experiment's sites causes the variation in predicted effect.

CONTENTS

TABLES

I. INTRODUCTION

When the Housing Assistance Supply Experiment (HASE) was planned, some observers were greatly concerned about the inflationary potential of earmarked cash allowances.* In principle, it seemed likely that allowance-induced demand shifts would force rents up, temporarily in a tight market and permanently in a loose market. In a tight market, increased demand was expected to raise rents only until the supply could be expanded through rehabilitation and new construction. In a loose market, it was presumed that rents would be discounted; increased demand would permanently erase the discounts.

Midway through the experiment, however, a fullscale allowance program has had no perceptible effect on rents, either in a tight market (Brown County, Wisconsin, whose main city is Green Bay) or in a loose market (St. Joseph County, Indiana, whose main city is South Bend).** One reason is clearly that the experimental program has generated less housing demand than anticipated. Another is that rents are much less sensitive, in the shortrun, to demand shifts than conventional market models suggest.

This report presents a theory of shortrun market adjustments to exogenous demand shifts consistent with the evidence from HASE. Combining the theory with data for Brown and St. Joseph counties, we

---

*The Housing Assistance Supply Experiment is studying two metropolitan housing markets (Brown County, Wisconsin, and St. Joseph County, Indiana) to help the U.S. Department of Housing and Urban Development assess the ability of housing allowances to enable low-income households obtain adequate housing. HASE was explicitly undertaken to measure the price effects of a fullscale housing allowance program, although it has other objectives as well. For the purpose, structure, and preliminary conclusions of the experiment, see the *Fourth Annual Report of the Housing Assistance Supply Experiment*, The Rand Corporation, R-2302-HUD, May 1978.

**For the evidence that the allowance program did not affect the price of rental housing services, see C. Lance Barnett and Ira S. Lowry, *How Housing Allowances Affect Housing Prices*, The Rand Corporation, R-2452-HUD, September 1979.

model the response of a rental market to demand shifts such as were generated by the experimental allowance program. We conclude that a 4 percent increase in demand due to an allowance program would at most cause rents to increase by one percent, given the initial conditions in the experimental sites. Only if the vacancy rate were close to zero would a demand increase produce substantial rent increases.

The theory has another implication: Even though rent is not sensitive to demand shifts, capital value is extremely so. A 4 percent increase in demand, if permanent, could cause capital value to rise as much as 20 percent. We cannot yet confirm such a result with longitudinal HASE data, but baseline market conditions in St. Joseph County's rental market offer partial confirmation. Preexperimental demand *decreases* (because of population losses) in central South Bend* caused rental property values to drop, such that they were 26 percent lower than elsewhere in the county, whereas rents for comparable dwellings are almost the same in the two submarkets.

CONDITIONS IN THE EXPERIMENTAL SITES

The HASE sites were chosen for their contrasting housing market and demographic characteristics. Because of rapid population growth, the 1973 rental vacancy rate was low in Brown County--about 4 percent, compared with a national rate of 6 percent. St. Joseph County had a segregated rental market, with nearly all the black population living in central South Bend. The 1974 rental vacancy rate in central South Bend was 13 percent, as against 6 percent for the remainder of the county.** The high vacancy rate resulted from job losses and

---

*Central South Bend includes all but the fringes of the city. It has three-fourths of South Bend's rental units and one-half of St. Joseph County's rental units.

**The vacancy rates for Brown and St. Joseph counties give the percentage of potential housing services not utilized, as measured by the rate of rent loss owing to vacancies. That is the correct measure of vacancies for this analysis, which follows Richard F. Muth's *Cities and Housing* (University of Chicago Press, 1969) and conceives of housing services as a homogeneous commodity. The conventional vacancy rates (vacant rental units as a percentage of all housing units) are 12.3 percent for central South Bend, 8.9 percent for the consequent household departures--especially from central South Bend--during the sixties and early seventies.

Brown County's increasing housing demand was met by sufficient new construction to keep the vacancy rate steady. Declining demand in St. Joseph County was not, on the other hand, matched by housing demolition, so vacancy rates rose far above those considered "normal," especially in the central South Bend submarket.

Table 1 shows how those events affected capital value and rent in central South Bend, the remainder of St. Joseph County, and Brown County. Because both capital value and rent have been standardized for intersite differences in the type and age of rental properties (see Appendix A), the entries are directly comparable. The unit of housing services is the amount provided by the average rental unit in the two counties. Value and rent are in 1974 dollars.

Table 1

CHARACTERISTICS OF RENTAL HOUSING IN BROWN AND ST. JOSEPH COUNTIES

Location	Average Vacancy Rate[a] (%)	Adjusted Average Capital Value ($/unit)	Adjusted Average Gross Rent ($/unit/yr)
Central South Bend	13.2	6,862	1,727
Rest of St. Joseph County	6.1	9,315	1,732
Brown County	4.2	12,316	1,764

SOURCE: HASE surveys of landlords at baseline, 1973 in Brown County, 1974 in St. Joseph County.

NOTE: Gross rent and capital value are adjusted to control for variation in building age and property type. Brown County figures are adjusted for price inflation during 1973-74 (see Appendixes A and B).

[a]Percent of rent lost due to vacancies.

---

rest of St. Joseph County, and 5.1 percent for Brown County. Note that the 6 percent average rental vacancy rate for the nation is based on the conventional definition, due to the lack of national data on rent losses.

Note that capital value varies inversely with market tightness, whereas rent is nearly the same in all three locations. St. Joseph County's housing surplus has not caused landlords to discount rents more than 2 percent, although it has caused the capital value of their properties to fall. In central South Bend, the value of rental housing is now 44 percent less than in Brown County.

THE OCCUPIED SUPPLY CURVE

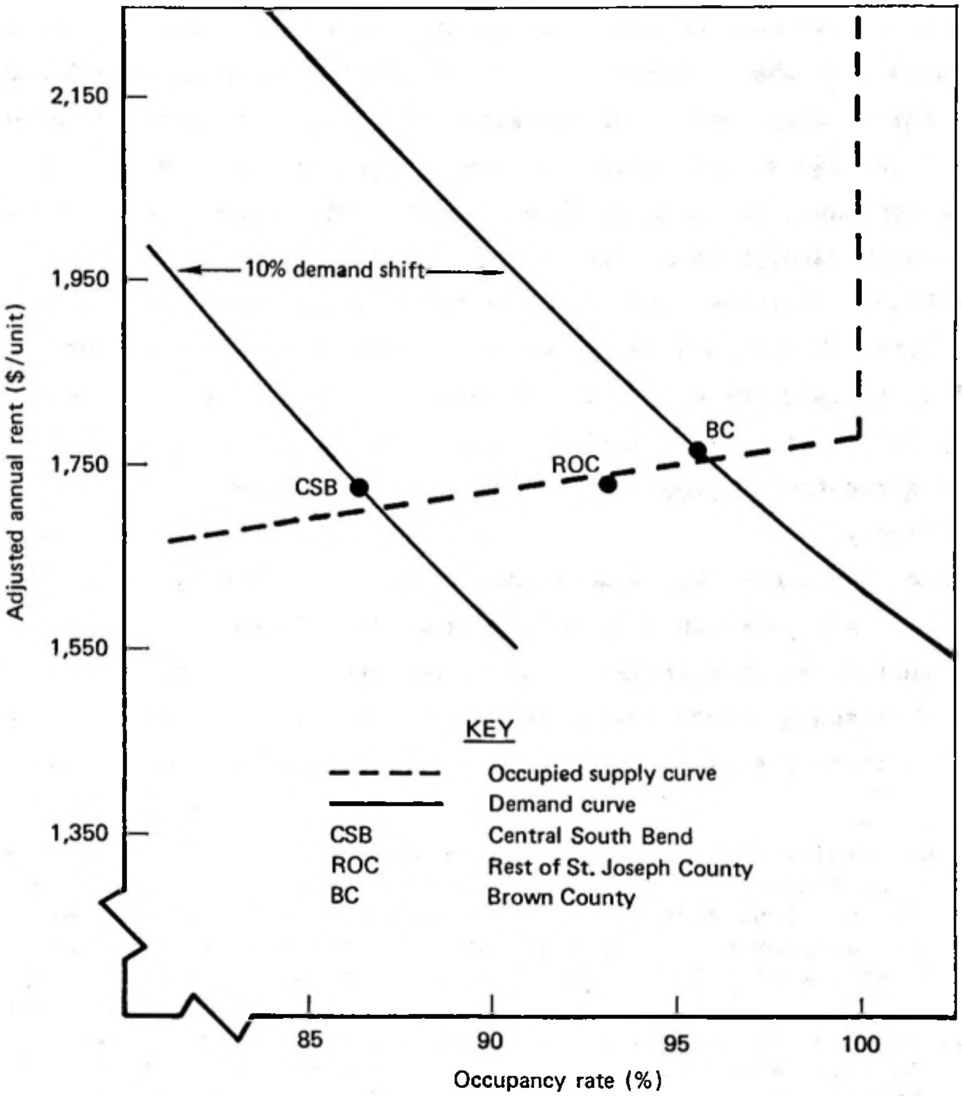
The curious insensitivity of rent to variations in market tightness is explained by the role vacancies play in shortrun market adjustments. When the demand curve shifts, there are two extreme possibilities. One is that the occupancy rate will remain constant, while rent changes bring demand into equilibrium with the occupied supply. The other is that rent will remain constant, while the occupancy rate changes. Actual market behavior falls between the extremes.

By occupancy rate we mean the fraction of housing services being consumed. (It equals 1.0 less the vacancy rate, when the vacancy rate is defined for units of housing services rather than for housing units.) When the demand curve shifts to the right, the occupancy rate can change in either of two ways, in the shortrun: (a) existing households can move from small housing units into larger ones, or (b) existing households can subdivide, so that each new one consumes more housing per capita. The traditional definition of occupancy rate--fraction of housing units occupied--measures only the second change. The definition of occupancy rate used in this report measures both.

Section II discusses how landlords set rents. For now, we represent the empirical relationship between the average occupancy rate and the market price of a unit of housing service by the *occupied supply curve*.* It is shown in the figure opposite, calibrated to the

---

*Such a curve is not a new concept in housing market analysis. It was proposed by Chester Rapkin, Louis Winnick, and David M. Blank in *Housing Market Analysis: A Study of Theory and Methods*, The Institute for Urban Land Use and Housing Studies, Columbia University, 1953, pp. 22-23; and elaborated in Blank and Winnick, "The Structure of the Housing Market," *Quarterly Journal of Economics*, Vol. 67, No. 2 (May 1953), pp. 181-208. More recently, but less explicitly, the



The occupied supply curve

---

occupied supply curve can be found in Frank de Leeuw and Nikanta F. Ekanem, "Time Lags in the Rental Housing Market," *Urban Studies*, Vol. 10 (1973), pp. 39-68 (to obtain the shortrun occupied supply curve from the market behavior equations in this article, hold housing stock constant and solve for the shortrun equilibrium relation between rent and occupancy rate). All those analyses point to monopolistic competition among landlords as the cause of the occupied supply curve. However, they do not develop the market theory as extensively as in Sec. II, nor do they trace the shortrun implications of demand shifts as sharply as in Sec. III.

three aggregate data points obtained from Table 1. The calibration indicates that the occupancy rate has a price elasticity of 3.4 until the stock is fully occupied, whereupon the elasticity drops to zero.* That is, on the sloped portion of the curve, a 3.4 percent increase in the occupancy rate (or roughly a 3.0 percentage-point decrease in the vacancy rate) corresponds to a 1.0 percent increase in rent.

The high price elasticity of occupied supply means that even substantial shifts in housing demand ordinarily affect rent very little. To illustrate, the figure shows aggregate demand curves for rental housing services passing through two points on the occupied supply curve that approximately correspond to central South Bend and Brown County.

The demand curves assume a price elasticity of 0.5, which is consistent with the range of estimates in the literature.** The 10 percent difference in the quantity of housing demanded (measured by the horizontal distance between the curves) is associated with an increase of only slightly more than 2 percent in the shortrun

---

*Regressing the natural logarithm of the occupancy rate on the natural logarithm of rent (using the data in Table 1) gives a slope of 3.39. The vertical part of the occupied supply curve is theoretical, not empirical, for none of the observed markets have zero vacancy rates. With the data available for this report we do not attempt anything more than a single estimate of the price elasticity of the occupancy rate. However, it is not necessarily the same for all occupancy rates. In particular, it seems reasonable that the sloped portion of the supply curve should turn upward as the vacancy rate closely approaches zero, making the curve meet the vertical portion smoothly, without a corner point. But whether the intersection of the sloped and vertical portions of the curve has a continuous derivative matters little for our analysis. What does matter is that in the range of observed occupancy rates, the curve slopes very gradually upward.

**For a recent literature review, see Stephen K. Mayo, *Theory and Estimation in the Economics of Housing Demand*, paper presented at the American Economic Association, Chicago, Illinois, 29-31 August 1978. The bulk of the estimates of the equilibrium price elasticity of demand lie between 0.2 and 0.8, centering on 0.5. Fortunately, as will be seen in Sec. III, our conclusions are not extremely sensitive to variations of the price elasticity of demand in that range. To avoid ambiguity about larger or smaller elasticities, this report defines the price elasticity of demand as a positive number--the percentage *decrease* in demand resulting from a 1.0 percent increase in price.

equilibrium rent. Correspondingly, a 1.0 percent shift in the demand curve would result in about only a 0.25 percent change in rent. Of course, if housing demand were to increase when the occupancy rate was already 100 percent, then shortrun supply would be inelastic. For every 1.0 percent shift of the demand curve, rent would increase by the inverse of the price elasticity of demand--2.0 percent.

The figure demonstrates that the rent increase caused by a demand increase depends on the vacancy rate existing before the demand shift. If the stock is less than fully utilized, the additional demand is met with vacant stock, and rents rise only slowly. When the stock is fully utilized, rents rise rapidly in response to increased demand.

Demand shifts such as those described above can occur for several reasons: for example, an exogenous change in population size, or a change in the average income of a stable population. And since the model uses occupancy rates rather than an absolute measure of the housing stock, a sudden change in the size of the stock is functionally equivalent to a demand shift.

OVERVIEW OF THE THEORY

The purpose of the theory presented here is to explain the occupied supply curve. Our estimates show the curve to be very flat (little rent change as the occupancy rate changes), meaning that the vacancy rate absorbs much of a demand shift.

Unlike the supply curve familiar to longrun equilibrium analysis, the occupied supply curve in shortrun equilibrium analysis depends on demand as well as on the costs of supply. Landlords set rents to maximize profits (rent received less operating costs); higher rents result in more vacancy losses, lower rents in less revenue from occupied units. Because vacancy losses at a given rent depend on aggregate demand, the resulting shortrun equilibrium (which occurs when no landlord has an incentive to choose a rent different from the average rent) depends on demand as well as on the fixed supply of housing services.

Our theory shows that shortrun equilibrium rents and occupancy rates depend on the price elasticity of aggregate demand for housing services, and on the additional price elasticity of demand for an individual landlord's housing services. The second elasticity stems from the increase in market share that would occur if the given landlord's rent was below the average. Although the first elasticity is a standard component of market analysis, the second is proposed here for the first time.

When the aggregate demand curve shifts, the shortrun equilibrium levels of rent and occupancy rate change, tracing out the occupied supply curve. The occupied supply curve is hence the locus of all the shortrun equilibrium points, each associated with a different aggregate demand. Even though it is convenient to identify shortrun equilibrium rent and occupancy rate as the intersection of the occupied supply and the aggregate demand curves, our theory proposes that causation runs the other way: Shortrun equilibria determine the occupied supply curve.

II. THEORY OF THE OCCUPIED SUPPLY CURVE

A rental housing submarket is in shortrun equilibrium when landlords cannot increase their profits (rent received less operating costs) by altering rent. As aggregate demand shifts, the different profit-maximizing rents (and the consequent average occupancy rates) define the occupied supply curve.*

This section first shows that shortrun equilibrium rent depends on the price elasticity of demand for an individual landlord's housing services. Then it separates that elasticity into the sum of the price elasticity of aggregate demand and a positive residual, and shows how the behavior of the additional price elasticity determines the sloped portion of the occupied supply curve. Finally, it compares the behavior of actual housing markets with what would occur if markets were monopolized or perfectly competitive.

SHORTRUN EQUILIBRIUM

Our analysis assumes that landlords establish rents that will maximize their current return, presuming that they have no control over average market rent. However, also assuming that all landlords behave the same, average rent changes in the direction of individual rents. Shortrun equilibrium occurs when individual landlords' rents do not differ from the average. Thus, where $\pi_i(R_i)$ is landlord i's current return as a function of his rent, and where R is average rent, shortrun equilibrium occurs when $\pi_i'(R_i) = 0$ and $R_i = R$.**

---

*Longrun equilibrium is the point on the occupied supply curve where capital value equals replacement cost, i.e., where there is no incentive for landlords to either expand or contract the supply of housing services.

**This definition of shortrun equilibrium is the classic solution of the monopolistic competition problem. In economics it is called the Cournot solution; in game theory, the Nash solution. The second-order condition, $\pi''(R_i) < 0$, showing that current return is maximized rather than minimized, is analyzed in Appendix C. The notation $f'(x)$ signifies the derivative of f with respect to x, and $f''(x)$ indicates the second derivative of f with respect to x.

The expected current return to a landlord equals rent received (full occupancy rent times the occupancy rate) less operating costs (utilities, management, maintenance, property tax, insurance, depreciation, and bad debts). Some operating costs are fixed, others vary with occupancy rate. The occupancy rate expected by an individual landlord equals the ratio of demand for his housing services D_i, which is a function of the individual landlord's rent R_i, to the supply H_i, which is fixed in the shortrun. Expected current return is thus

$$\pi_i(R_i) = R_i H_i \left[\frac{D_i(R_i)}{H_i}\right] - bH_i - cH_i \left[\frac{D_i(R_i)}{H_i}\right], \qquad (2.1)$$

where R_i = rent charged by landlord i per unit of housing services,

$\pi_i(R_i)$ = expected current return to landlord i as a function of his rent,

$D_i(R_i)/H_i$ = expected occupancy rate of housing services provided by landlord i (ratio of demand D_i to supply H_i),

b = fixed operating cost per unit of potential housing services (does not vary with occupancy rate),

c = variable operating cost per unit of housing services sold.

A landlord's expected current return is maximized when the derivative of current return with respect to rent equals zero:

$$\pi_i'(R_i) = D_i(R_i) + \left[R_i - c\right]\left[D_i'(R_i)\right] = 0. \qquad (2.2)$$

That condition can be rewritten as

$$-D_i'(R_i)\left[\frac{R_i}{D_i(R_i)}\right] = \frac{1}{1 - c/R_i} \,. \qquad (2.3)$$

Then, recognizing the left side as the definition of the price elasticity of demand for the individual landlord's housing services, and recalling that individual rents equal average rents in equilibrium, we define shortrun equilibrium rent by a pair of equations

$$S_i(R_i) = \frac{1}{1 - c/R_i} \qquad (2.4)$$

$$R_i = R,$$

where

$$S_i(R_i) = \frac{-D_i'(R_i)R_i}{D_i(R_i)},$$

or the price elasticity of demand for a landlord's housing services (percentage decrease in demand per 1.0 percent increase in price), and R = average rent.

PRICE ELASTICITY OF DEMAND

If one landlord asks a rent different from the average, the demand for his housing services will differ from his proportional share of aggregate demand, being greater if what he asks is lower than the average, and smaller if what he asks is higher. That is,

$$D_i(R_i) \gtreqless \left[\frac{H_i}{H}\right] D(R) \quad \text{if} \quad R_i \lesseqgtr R, \qquad (2.5)$$

where $D_i(R_i)$ = demand for a landlord's housing services at his rent R_i,

$D(R)$ = aggregate demand for housing services at average rent R,

H_i = individual landlord's supply of housing services (fixed in the shortrun),

H = aggregate supply of housing services (fixed in the shortrun).

Equation (2.5) remains true even if aggregate demand at average rent is replaced with what it would be at landlord i's rent, because his demand responds not only to the same tradeoffs between housing and other goods as aggregate demand but also to changes in market share caused by price differentials. That is,

$$D_i(R_i) \gtreqless \left[\frac{H_i}{H}\right] D(R_i) \quad \text{if} \quad R_i \lesseqgtr R, \tag{2.6}$$

where $D(R_i)$ = aggregate demand as a function of rent, evaluated at R_i, the individual landlord's rent.

Accordingly, the demand for an individual landlord's housing services decomposes into his proportional share of aggregate demand if all landlords charged his same rent, and a residual that is a decreasing function of his rent (zero if his rent equals average rent):

$$D_i(R_i) = \left[\frac{H_i}{H}\right] D(R_i) + M_i(R_i), \tag{2.7}$$

where $M_i(R_i)$ = difference between the demand for an individual landlord's housing services and his proportional share of aggregate demand if all landlords charged his rent ($M(R_i) = 0$ when $R_i = R$ and $M_i'(R_i) < 0$).

To transform the demand decomposition into a price elasticity decomposition, we first differentiate Eq. (2.7) with respect to R_i, then multiply both sides of the resulting equation by $R_i/D_i(R_i)$. Finally, we multiply both numerator and denominator of the first term on the right side of the equation by $D(R_i)$:

$$\frac{D_i'(R_i)R_i}{D_i(R_i)} = \left[\frac{D'(R_i)R_i}{D(R_i)}\right]\left[\frac{[H_i/H]D(R_i)}{D_i(R_i)}\right] + \frac{M_i'(R_i)R_i}{D_i(R_i)}. \tag{2.8}$$

In more compact notation,

$$S_i(R_i) = S(R_i)\left[\frac{[H_i/H]D(R_i)}{D_i(R_i)}\right] + T, \tag{2.9}$$

where

$$S(R_i) = \frac{-D'(R_i)R_i}{D(R_i)},$$

or the price elasticity of aggregate demand for housing services (percentage decrease in aggregate demand per 1.0 percent increase in price), and

$$T = \frac{-M_i'(R_i)R_i}{D_i(R_i)} ,$$

or the additional price elasticity of demand for an individual landlord's supply of housing services ($T > 0$ because $M_i'(R_i) < 0$).

Because $D_i(R_i) = [H_i/H]D(R_i)$ when $R_i = R$, the simplest form of the decomposition is

$$S_i(R_i) = S(R_i) + T \quad \text{if} \quad R_i = R. \tag{2.10}$$

Equation (2.10) shows that at average rent, the price elasticity of demand for an individual landlord's housing services S_i equals the sum of the price elasticity of the aggregate demand for housing services S and a positive residual T, which is the additional price elasticity of demand for a landlord's housing services.

The price elasticity of aggregate demand is written as a function of rent ($S = S(R)$) because, in general, the price elasticity varies along the demand curve. We assume that the price elasticity of aggregate demand is an increasing function of price, $S'(R) > 0$, so that the second-order condition for profit maximization will be met (see Appendix C). That condition is not strongly restrictive; any demand curve that is less convex than a rectangular hyperbola, e.g., a linear demand curve, satisfies it (again, see Appendix C).

The additional price elasticity of demand for an individual landlord's housing services T is the most important behavioral component of our housing market theory. It decreases as the average occupancy rate F increases, because in a tighter market, people comparison-shop less for fear of losing a good deal once found. Given the assumption that $S'(R) > 0$, Eq. (2.14) will show that the occupied supply curve has a positive slope (and hence that rent increases as market conditions tighten) if and only if the additional price elasticity T decreases as average occupancy rate F increases. That result makes $T'(F) < 0$ the key behavioral relationship in our theory. Readers who are not persuaded by the comparison-shopping argument are free to treat $T'(F) < 0$ as an implication of the evidence offered on pp. 4-7 that the occupied supply curve has a positive slope.

PRICE ELASTICITY OF THE OCCUPANCY RATE

Equation (2.4) defines shortrun equilibrium in terms of the total price elasticity of demand for an individual landlord's housing services. Equation (2.10) separates that elasticity into the sum of the price elasticity of aggregate demand and the additional price elasticity of an individual landlord's demand. Combining the two equations, recognizing that in equilibrium, individual rent equals average rent, and applying the conclusion that the additional price elasticity is a function of average occupancy rate, the shortrun equilibrium condition becomes*

$$S(R) + T(F) = \frac{1}{1 - c/R}\,, \tag{2.11}$$

where $S(R)$ = price elasticity of aggregate demand as a function of average rent R,

$T(F)$ = additional price elasticity of an individual landlord's demand as a function of the average occupancy rate F,

c = variable operating costs per unit of housing service under full occupancy.

Equation (2.11) defines the occupied supply curve: the relation between rent R and occupancy rate F as shifts occur in aggregate demand. (The link to aggregate demand is established by the identity that defines the average occupancy rate F as the ratio of aggregate demand to supply--$D(R)/H$.)

---

*As a guide to the sizes of the variables in this equation, we anticipate the estimate in Sec. III that $c/R \cong 0.18$ (i.e., that variable operating costs under full occupancy are about 18 percent of rent) and conclude that $1/[1 - c/R] \cong 1.2$. Based on the literature's estimate of the equilibrium price elasticity of aggregate demand $S \cong 0.5$, the additional price elasticity of demand for an individual landlord is $T \cong 0.7$.

To show that the theoretical occupied supply curve has a positive slope, and hence is consistent with observed housing market behavior, we show that the price elasticity of the occupancy rate is positive. The elasticity is the percentage change in occupancy rate resulting from a 1.0 percent increase in rent, moving along the locus of shortrun equilibrium points:

$$Z = \frac{F'(R)R}{F(R)}, \tag{2.12}$$

where Z = price elasticity of the occupancy rate,
$F(R)$ = occupancy rate as a function of rent, as implicitly defined by Eq. (2.11).

Differentiating Eq. (2.11) with respect to rent R yields

$$S'(R) + T'(F)F'(R) = \frac{-c/R^2}{[1 - c/R]^2}, \tag{2.13}$$

and solving for the price elasticity of the occupancy rate gives

$$Z = \frac{S'(R)R + \dfrac{c/R}{[1 - c/R]^2}}{-T'(F)F}. \tag{2.14}$$

Under the conditions $S'(R) > 0$ and $T'(F) < 0$ discussed earlier, the price elasticity of the occupancy rate is always positive, so the occupied supply curve slopes upward.

MONOPOLY VERSUS COMPETITION

Housing markets are not monopolies because ownership is extremely fragmented. For example, Brown County had 4,800 different owners of 6,500 rental properties in 1973, and St. Joseph County had 6,400 different owners of 9,800 rental properties in 1974. Neither are markets perfectly competitive, because housing units offer different quantities of housing service, and tenants have imperfect information about those services. Only by inspecting a dwelling can one know how much service it offers and hence the price per unit of service.

Housing market behavior is in fact monopolistically, or imperfectly, competitive--lying between perfect competition and monopoly. Both the price elasticity of occupied supply and the equilibrium price of housing services at a given aggregate demand are smaller than under monopoly and larger than under perfect competition. Since the effect of a demand shift on rent diminishes as the price elasticity of occupied supply increases (i.e., as the slope of the occupied supply curve steepens, plotting rent on the vertical axis), we conclude that a demand increase under monopolistic competition causes rent increases larger than under monopoly and smaller than under perfect competition.

If the housing market were a monopoly, the profit-maximizing rent would be independent of a multiplicative shift* in aggregate demand, the occupied supply curve would be horizontal, and the price elasticity of occupied supply would be infinite. Since one landlord would control the market, the price elasticity of demand for the individual landlord's housing services would be identical to the price elasticity of aggregate demand. In other words, the additional price elasticity of demand T would be zero, and Eq. (2.11) defining shortrun equilibrium would become $S(R) = 1/[1 + c/R]$. Because the function $S(R)$ is not affected by multiplicative shifts in aggregate demand,** the equilibrium rent under monopoly would not be affected by such shifts. Note that our argument also implies that rent under monopoly price would be more than under monopolistic competition, for as T becomes smaller in Eq. (2.11), $S(R)$ must increase; and because we assume that $S'(R) > 0$, R must also increase.

If the housing market were perfectly competitive (with both landlords and tenants having full information on quantities and prices), the vacancy rate would always be zero. The occupied supply curve would be identical to the vertical shortrun supply curve, and the

---

*A shift in aggregate demand that occurs when demand at all price levels is multiplied by the same factor.

**Multiplying demand by a constant factor causes the derivative of demand with respect to rent to be multiplied by the same factor, leaving the price elasticity of demand, $S(R) = -D'(R)R/D(R)$, unchanged.

price elasticity of occupied supply would be zero. The additional price elasticity of demand for an individual landlord's housing services would be infinite. It follows from Eq. (2.11) and the assumption $S'(R) > 0$ that as T grows, R shrinks. Before T becomes infinite, the constraint that demand cannot exceed supply, $D(R) \leq H$, will of course bind, and Eq. (2.11) will no longer apply. However, rent is a minimum when the constraint binds, so the argument shows that rent under perfect competition would be less than under monopolistic competition.

III. MARKET RESPONSE TO DEMAND SHIFTS

Price elasticities of 0.5 for demand and 3.4 for occupancy rate imply that when aggregate demand increases 1.0 percent,* rent increases only 0.26 percent, while the occupancy rate increases 0.87 percent. Capital value can increase as much as 5.0 percent. In general, the response of rent and occupancy rate to a demand shift depends only on the two price elasticities. The response of capital value depends on operating costs, initial value, and the duration of the demand shift, as well as on the elasticities.

RESPONSES OF OCCUPANCY RATE AND RENT

Since the intersection of the occupied supply curve with the demand curve gives the shortrun equilibrium occupancy rate and rent, it is not surprising that the percentage changes in those variables per 1.0 percent change in demand should depend on the price elasticities of demand and occupancy rate.

To analyze demand shifts, demand can be treated as the product of a scale factor and a reference level of demand, then normalized by supply to yield the occupancy rate:

$$F = \frac{Nd(R)}{H} , \qquad (3.1)$$

where $Nd(R) = D(R)$ = aggregate demand as a function of average rent R, expressed as the product of a scale factor N and a reference demand curve $d(R)$,

H = total potential supply of housing services (fixed in the shortrun),

F = occupancy rate (fraction of potential housing services consumed).

---

*A 1.0 percent increase in aggregate demand means that demand at any given price is multiplied by 1.01--i.e., that the entire demand curve shifts 1.0 percent to the right.

Equation (3.1) defines occupancy rate as a function of rent along the demand curve. Equation (2.11), on the other hand, implicitly defines occupancy rate as a function of rent along the occupied supply curve, i.e., along the locus of alternative shortrun equilibria; that is,

$$F = g(R), \tag{3.2}$$

where $g(R)$ = occupancy rate as a function of rent along the occupied supply curve.

To determine the effect of a demand shift, let Eqs. (3.1) and (3.2) define rent R and occupancy F as implicit functions of the demand scale factor N. We seek the response elasticities*

$$\varepsilon(R) = \frac{R'(N)N}{R} \tag{3.3}$$

and

$$\varepsilon(F) = \frac{F'(N)N}{F}, \tag{3.4}$$

where $\varepsilon(R)$ = percentage change in rent per 1.0 percent increase in demand,

$\varepsilon(F)$ = percentage change in the occupancy rate per 1.0 percent increase in demand,

as functions of the price elasticities

$$S = \frac{-d'(R)R}{d(R)} \tag{3.5}$$

---

*Note that if for some reason (e.g., earthquake, fire, flood, hurricane) the housing supply H is not constant in the shortrun, the response elasticities should be defined with respect to $J = N/H$ rather than N, because rent and occupancy rate respond to changes in demand relative to supply rather than to changes in the absolute level of demand. The use of $J = N/H$ instead of N in Eqs. (3.4) and (3.5) leads to the same conclusion.

and

$$Z = \frac{g'(R)R}{g(R)}, \tag{3.6}$$

where S = price elasticity of demand (defined as the (positive) percentage decrease in demand per 1.0 percent increase in price),

Z = price elasticity of the occupancy rate (moving along the occupied supply curve).

We therefore differentiate Eqs. (3.1) and (3.2) with respect to the scale factor N to find

$$F'(N) = \frac{1}{H}\left[d(R) + Nd'(R)R'(N)\right] \tag{3.7}$$

and

$$F'(N) = g'(R)R'(N). \tag{3.8}$$

Multiplying by N/F and rearranging terms, the two equations become

$$\frac{F'(N)N}{F} = \frac{Nd(R)}{FH}\left[1 - \left(\frac{-d'(R)R}{d(R)}\right)\left(\frac{R'(N)N}{R}\right)\right] \tag{3.9}$$

and

$$\frac{F'(N)N}{F} = \left(\frac{g'(R)R}{F}\right)\left(\frac{R'(N)N}{R}\right). \tag{3.10}$$

Substituting Eqs. (3.1) through (3.6) into Eqs. (3.9) and (3.10)--noting that $Nd(R)/FH = 1$--we obtain

$$\varepsilon(R) = 1 - S\varepsilon(R) \tag{3.11}$$

and

$$\varepsilon(R) = Z\varepsilon(R). \tag{3.12}$$

Solving Eqs. (3.11) and (3.12) yields the responses of rent and occupancy rate to a 1.0 percent increase in demand, as functions of the price elasticities of demand and the occupancy rate:

$$\varepsilon(R) = \frac{1}{Z + S} \tag{3.13}$$

and

$$\varepsilon(F) = \frac{Z}{Z + S} . \tag{3.14}$$

For $S = 0.5$ and $Z = 3.4$, $\varepsilon(R) = 0.26$ and $\varepsilon(F) = 0.87$, as noted earlier. Those conclusions are only slightly sensitive to S, the equilibrium price elasticity of demand. As S ranges from 0.2 to 0.8 (the range of estimates in the literature), $\varepsilon(R)$ varies only from 0.28 to 0.24, and $\varepsilon(F)$ from 0.95 to 0.82. The value of $\varepsilon(R) = 0.26$ is, however, sensitive to Z, the price elasticity of the occupancy rate. If Z was half our estimate of 3.4, then $\varepsilon(R)$ would be 0.45, almost twice our result of 0.26.

RESPONSE OF CAPITAL VALUE

Capital value equals the discounted sum of anticipated current returns, defined as in Eq. (2.1). Equation (3.15) separates current returns influenced by the shortrun demand shift (through year n) from those that occur after the shortrun demand shift has been counteracted (either by reversing the demand shift or by changing supply):

$$V = \int_{t=0}^{n} [RF - b - cF]e^{-rt}\, dt + \int_{t=n}^{\infty} [RF - b - cF]e^{-rt}\, dt, \tag{3.15}$$

where V = capital value of a unit of housing services,

b = fixed operating cost,

c = variable operating cost under full occupancy,

r = real opportunity cost rate,

n = years the demand shift--relative to supply--is expected to last.

Letting Eqs. (3.1), (3.2), and (3.15) define R, F, and V as implicit functions of the demand scale factor N, the response elasticity for capital value is

$$\varepsilon(V) = \frac{V'(N)N}{V}, \tag{3.16}$$

where $\varepsilon(V)$ = percentage change in capital value per 1.0 percent increase in demand.

Differentiating Eq. (3.15) with respect to N (assuming that R and F respond to changes in N only before year n) yields

$$V'(N) = \int_{t=o}^{n} \left[R'(N)F + [R - c]F'(N)\right] e^{-rt}\, dt. \tag{3.17}$$

Evaluating the integral, multiplying both sides of the equation by N/V, and multiplying numerator and denominator of the right side by $[R - c]F$ gives

$$\frac{V'(N)N}{V} = \left[\left(\frac{R'(N)N}{R}\right)\left(\frac{R}{R - c}\right) + \frac{F'(N)N}{F}\right]\left[\frac{[R - c]F}{rV}\right]\left[1 - e^{-rn}\right]. \tag{3.18}$$

From Eq. (3.15), $V = [RF - b - cF]/r$, so $[R - c]F = rV + b$ (before taking demand shifts into account). Substituting the elasticities in Eqs. (3.3), (3.4), (3.13), (3.14), and (3.16), we therefore conclude that

$$\varepsilon(V) = \left[\frac{Z + 1/[1 - c/R]}{Z + S}\right]\left[1 + \frac{b}{rV}\right]\left[1 - e^{-rn}\right]. \tag{3.19}$$

The response elasticity for capital value $\varepsilon(V)$ thus depends on the operating costs b and c and the real opportunity cost rate r. Tables 2 and 3 provide relevant information from the HASE surveys. Table 2 shows that the annual total operating cost is \$1,220 per unit of housing services (taken to be the amount provided annually by the average rental unit in either county). We have not yet analyzed how much of the operating cost is fixed or how much varies

Table 2

ANNUAL OPERATING COSTS OF RENTAL HOUSING IN
BROWN AND ST. JOSEPH COUNTIES

Component of Operating Cost	Annual Amount ($) per Unit			
	Central South Bend	Rest of St. Joseph County	Brown County	All Locations
Utilities	477	410	361	404
Management	155	159	152	155
Maintenance	286	254	234	253
Property tax	199	173	298	239
Insurance	56	58	41	49
Depreciation[a]	54	84	129	97
Bad debts[b]	42	20	10	21
Total	1,269	1,158	1,225	1,218

SOURCE: HASE surveys of landlords at baseline, 1973 in Brown County, 1974 in St. Joseph County.

NOTE: Brown County data for 1973 are adjusted for price inflation during 1973-74; data in all locations are adjusted for differences in property age and size (see Appendixes A and B).

[a]Estimate of depreciation in property value caused by the property's aging one year (deterioration of capital plus effect of increased maintenance costs). Does not include changes in property value caused by changes in market conditions.

[b]Rent loss from bad debts.

with the occupancy rate. However, it seems reasonable that three-fourths of utility costs vary with occupancy rate, and that little else does. Since utility costs are $400 per year, CF = 300 and b = 920. Then, because the average occupancy rate in Brown and St. Joseph counties is 0.93, we conclude that c = 320.

The rate of real opportunity cost equals real return to capital divided by capital value. Real return is rent less vacancy loss and operating cost. The estimated rate varies from a low of 3.4 percent in central South Bend to a high of 5.0 percent in the rest of St. Joseph County. However, the likely estimation errors for the numerator and denominator of the ratio are too large to imply certain

Table 3

REAL OPPORTUNITY COST OF CAPITAL FOR RENTAL HOUSING
IN BROWN AND ST. JOSEPH COUNTIES

Item	Central South Bend	Rest of St. Joseph County	Brown County	All Locations
Annual Amount ($) per Unit				
Gross rent[a]	1,727	1,732	1,764	1,746
Vacancy rent loss[b]	228	106	74	123
Operating cost[c]	1,269	1,158	1,225	1,218
Real return[d]	230	468	465	405
Amount ($) per Unit or Fraction				
Capital value[e]	6,862	9,315	12,316	10,088
Rate of real return[f]	0.034	0.050	0.038	0.040

SOURCE: HASE surveys of landlords at baseline, 1973 in Brown County, 1974 in St. Joseph County.

NOTE: Brown County data for 1973 are adjusted for price inflation during 1973-74; data in all locations are adjusted for differences in property age and size (see Appendixes A and B).

[a]Contract rent plus direct tenant payment for utilities and repairs.

[b]Contract rent lost due to vacancies plus direct tenant payments not made because of utilities not consumed during vacancies.

[c]See Table 2.

[d]Gross rent less vacancy rent loss and operating cost.

[e]Owner estimate of property value at time of baseline survey, adjusted to mid-1974.

[f]Annual real return as fraction of capital value.

variation in the opportunity cost rate; this analysis therefore uses the overall average $r = 0.04$ (refer to Table 3).

Using estimates of $b = 920$, $c = 320$, and $r = 0.04$, Table 4 shows how the response elasticity for capital value varies with the number of years the demand shift is expected to last n and the initial capital value V. Not surprisingly, the response elasticity is greater

Table 4

PERCENTAGE INCREASE IN CAPITAL VALUE PER 1.0 PERCENT INCREASE IN DEMAND

Expected Duration of Relative Demand Shift (yrs)	Central South Bend	Rest of St. Joseph County	Brown County
5	0.93	0.74	0.61
10	1.70	1.36	1.12
20	2.81	2.26	1.87
40	4.13	3.29	2.72
∞	5.16	4.11	3.40

SOURCE: Equation (3.19) with $S = 0.5$, $Z = 3.4$, $b = 920$, $c = 320$, $r = 0.04$, $R = 1{,}727$ and $V = 6{,}862$ in central South Bend, $R = 1{,}732$ and $V = 9{,}315$ in the rest of St. Joseph County, and $R = 1{,}764$ and $V = 12{,}316$ in Brown County.

the longer the expected demand shift and the lower the initial capital value.

If a demand shift of 1.0 percent occurs in a loose market where capital value has been heavily discounted (as in central South Bend) and if the shift is expected to be permanent, then capital value will increase as much as 5.0 percent. However, under more realistic expectations about how long the demand shift will last (before being counteracted by a supply response)--say, 5 to 10 years--capital value increases 0.60 to 1.70 percent for each 1.0 percent increase in demand.

IV. CONCLUSIONS

According to our monopolistic competition theory of the housing market, shortrun equilibrium depends on two price elasticities of demand: the price elasticity of aggregate demand, a familiar concept; and the additional price elasticity of demand for an individual landlord's housing services, a new concept developed here. Based on an estimated aggregate elasticity of 0.5, the additional elasticity is about 0.7, giving 1.2 as the total price elasticity of demand for an individual landlord's housing services.

Because the price elasticity of demand for an individual landlord is higher than the price elasticity of aggregate demand, the market's equilibrium rent is less than under a monopoly. On the other hand, the additional price elasticity of demand cannot be infinite because of imperfect information, so that the market's equilibrium rent is greater than under perfect competition.

Under monopolistic competition, the effect of a demand shift on rent is also between that under a monopoly and that under perfect competition. Monopolistic competition causes both rent and occupancy rate to increase when the aggregate demand curve shifts to the right. Under monopoly, only the occupancy rate would change, and under perfect competition, only rent would change. In short, the monopoly aspect of monopolistic competition is what causes vacancy rates (i.e., occupancy rates of less than 100 percent), and the competitive aspect of monopolistic competition is what causes rents to be lower in loose housing markets than in tight ones.*

---

*The conclusion that market behavior under monopolistic competition lies between that under monopoly and that under perfect competition requires some restrictions on the behavior of housing demand. We assume that price increases cause increases in the price elasticities of both aggregate demand as a function of average rent and additional demand for an individual landlord's housing services as a function of his rent; and that the price elasticity of additional demand decreases as average occupancy rate increases (see Sec. III and Appendix C). Those restrictions are consistent with the observed housing market behavior--a positively sloped occupied supply curve.

Rent is not very sensitive to demand shifts. Our empirical estimates show that the vacancy rate absorbs most of the shock caused by aggregate demand shifts, leaving rent little affected. Specifically, a 1.0 percent increase in demand causes the occupancy rate to increase 0.87 percent, but rent to increase only 0.26 percent.

In contrast, capital value is sensitive to demand shifts. The changes in both rent and vacancy loss are capitalized, so here the vacancy rate increases rather than absorbs demand shock. A 1.0 percent increase in demand can cause capital value to increase as much as 5.0 percent if the demand shift is permanent relative to supply and if the value increase starts from a low capital value in a loose market. However, if the relative demand shift is expected to last only five years, and if capital value is high in a tight market, then capital value increases only 0.6 percent per 1.0 percent increase in demand.

To apply our theory to HASE, we must first estimate the size of the demand shift caused by the experimental program. We have constructed an *a fortiori* argument from estimates assuming that the income elasticity of housing demand is 0.5--higher than HASE data show it to be.[*] Therefore, our estimate that demand increased by 2.0 to 4.0 percent because of the program is an upper bound on the actual program-induced shift, and our estimates of the effects are also upper bounds.

The estimates in Table 5 of the additional demand caused by the allowance program are only approximations. It is clear, however, that the program-induced demand is much lower than preexperimental expectations. The participation rate of eligibles, the income elasticity of demand, and the difficulty of upgrading substandard housing units have all proved less than expected.

Applying the response elasticities for rent, occupancy rate, and capital value derived in Sec. III, the predicted effects of the demand shifts are as follows: Rent increases only 0.6 to 1.0 percent,

---

[*] John E. Mulford's *Income Elasticity of Housing Demand* (The Rand Corporation, R-2449-HUD, July 1979) finds that the income elasticity of demand for renters is only 0.19.

Table 5

ESTIMATED EFFECTS OF THE ALLOWANCE PROGRAM ON RENTAL HOUSING

Location	Households Receiving Allowance Payments[a] (%)	Increase in Recipient Income Caused by Allowances[b] (%)	Housing Units Removed from Market[c] (%)	Increase in Aggregate Demand for Housing Services[d] (%)
Central South Bend	17.2	32.4	1.0	3.8
Rest of St. Joseph County	12.2	28.6	0.5	2.2
Brown County	13.3	21.4	1.3	2.7
All locations	14.0	25.8	1.0	2.8

SOURCE: HASE baseline surveys, housing allowance office client characteristics file, and estimation procedures defined by the following notes.

[a] Recipient households at the end of the program's third year (June 1977 in Brown County, December 1977 in St. Joseph County) as percent of all households at baseline (1973 in Brown County, 1974 in St. Joseph County).

[b] Mean allowance payment as percent of mean gross income of recipients without the allowance payment, at the end of the program's third year.

[c] Substandard rental units removed from the market (estimated by the three-year total number of units that failed a program inspection and whose occupants then moved, found standard housing, and began receiving payments), as percent of all rental units at baseline.

[d] First column (divided by 100) times second column times 0.5 (an estimate of the income elasticity of housing demand) plus third column (on the grounds that removing supply effectively increases the demand for the remaining supply).

occupancy rate increases 1.9 to 3.3 percent, and capital value increases 1.6 to 6.5 percent (see Table 6). Capital value was predicted with the 10 years of the experimental program as the expected duration of the demand shift, except that for Brown County's tight market, we assumed that supply would respond to the demand shift within five years.

Table 6

MAXIMUM SHORTRUN RESPONSE TO THE ALLOWANCE PROGRAM

Location	Increase (%)			
	Rental Demand	Rent	Occupancy Rate	Capital Value
Central South Bend	3.8	1.0	3.3	6.5
Rest of St. Joseph County	2.2	0.6	1.9	3.0
Brown County	2.7	0.7	2.3	1.6

SOURCE: Table 5 and Eqs. (3.13), (3.14), and (3.19) with $S = 0.5$, $Z = 3.4$, and $n = 10$ in St. Joseph County and $n = 5$ in Brown County.

NOTE: The estimated demand shift, and hence the estimated market effects, are upper bounds (see accompanying text). Comparisons of predicted with actual changes in both counties must control for background price inflation and the effect of any demand changes besides those owing to the allowance program.

The limitations of the analysis are first, that we used a high (0.5) income elasticity of demand, and second, that we restricted ourselves to rental housing. To the extent that an increase in rental demand causes owner-occupied housing to become rental, the effective demand shift will decrease, thereby reducing the effect on market conditions.

Those limitations add force to the conclusion that a housing allowance program does not cause rent increases that are relevant to housing policy. The predicted rent increases range from 0.6 percent in Brown County to 1.0 percent in central South Bend; the theory's limitations imply that actual increases are even smaller.

Our theory explicates observed market behavior. It explains why vacancies exist, and it shows why rents change less than capital values as a consequence of demand shifts. Applying the theory to HASE shows why the allowance program has not perceptibly increased rents--vacancy rates at baseline were large enough to absorb most of the small demand shift.

Considerable testing and extending of the theory obviously remain. Directions of further research lie, for example, in examining the relationship between occupancy rates and rents using additional data, and in extending the theory by analyzing supply adjustment to longrun equilibrium.

Appendix A
CONTROLLING FOR PROPERTY AGE AND SIZE

Housing deteriorates with age; average unit size decreases as property size increases. Therefore, for rent per unit to measure the price of housing services and capital value per unit to measure the price of housing capital, property age and size must be taken into account.

Rental units in central South Bend are, on the average, older than those in the rest of St. Joseph County or in Brown County. Rental units in the rest of St. Joseph County are more often single-family houses and less often duplexes, triplexes, or quadplexes than in Brown County. The differences in the distribution of rental units by property age and size (detailed in Table A.1) are large enough to significantly affect the per-unit averages of gross rent and capital value.

Table A.2 reports average gross rent per unit and average capital value per unit for different property ages and sizes. The adjusted averages control for age and size variations by weighting the stratum averages equally. Comparing the adjusted averages with the usual averages shows that the adjustment has little effect in Brown County and opposite effects in the two parts of St. Joseph County. Controlling for property age and size raises average rent and value in central South Bend and lowers average rent and value in the rest of St. Joseph County. (All the averages in this report are adjusted by the same method as rent and value per unit are adjusted in Table A.2.)

Table A.3 reports the sizes of the samples analyzed in this report.* The sample covers all age-size strata in all locations, and the samples in each location are ample. Some cells are small, however, so that comparing cells within Table A.2 requires caution.

---

*The sample elements are rental properties, not rental units. In addition, the sample consists only of "regular" rental properties; excluded are mobile home, rooming house, farm, and federally subsidized properties.

Table A.1

DISTRIBUTION OF UNITS ON RENTAL RESIDENTIAL PROPERTY IN BROWN AND ST. JOSEPH COUNTIES

Size of Property and Year Built	Percent of All Units		
	Brown County	Central South Bend	Rest of St. Joseph County
1 Unit			
Post-1944	3.7	3.8	10.8
1915 to 1944	7.7	13.5	16.7
Pre-1915	3.8	16.9	6.3
2-4 Units			
Post-1944	21.1	0.6	3.2
1915 to 1944	22.8	14.3	8.6
Pre-1915	20.8	41.1	17.9
5+ Units			
Post-1944	16.0	3.2	20.4
1915 to 1944	2.5	1.1	13.3
Pre-1915	1.6	5.5	2.8
Total	100.0	100.0	100.0

SOURCE: HASE surveys of landlords at baseline, 1973 in Brown County, 1974 in St. Joseph County.

Table A.2

GROSS RENT AND CAPITAL VALUE OF RENTAL HOUSING IN BROWN AND ST. JOSEPH COUNTIES

Size of Property and Year Built	Annual Rent ($) per Unit: Brown County	Annual Rent ($) per Unit: Central South Bend	Annual Rent ($) per Unit: Rest of St. Joseph County	Capital Value ($) per Unit: Brown County	Capital Value ($) per Unit: Central South Bend	Capital Value ($) per Unit: Rest of St. Joseph County
1 Unit						
Post-1944	2,151	1,783	1,896	17,298	7,744	11,191
1915 to 1944	1,910	2,022	1,789	13,743	8,208	9,778
Pre-1915	1,702	1,840	1,927	12,454	6,850	10,006
2-4 Units						
Post-1944	2,171	1,551	1,808	16,971	6,142	10,718
1915 to 1944	1,551	1,461	1,535	9,849	6,508	8,463
Pre-1915	1,448	1,377	1,300	8,703	4,951	6,714
5+ Units						
Post-1944	1,984	2,829	2,568	14,661	10,206	14,933
1915 to 1944	1,443	1,332	1,445	8,031	5,329	5,814
Pre-1915	1,515	1,348	1,320	9,131	4,563	6,213
Average[a]	1,783	1,615	1,799	12,507	6,290	9,728
Adjusted average[b]	1,764	1,727	1,732	12,316	6,862	9,315
Adjustment (%)	-1.1	6.9	-3.7	-1.5	9.1	-4.2

SOURCE: HASE surveys of landlords at baseline, 1973 in Brown County, 1974 in St. Joseph County.

NOTE: Brown County data for 1973 are adjusted for price inflation during 1973-74 (see Appendix C).

[a] Average of column entries weighted by the distribution of units in Table A.1.

[b] Average of column entries weighted equally.

Table A.3

NUMBER OF RENTAL PROPERTIES IN ANALYSIS SAMPLE

Size of Property and Year Built	Brown County	Central South Bend	Rest of St. Joseph County
1 Unit			
Post-1944	179	35	107
1915 to 1944	361	124	180
Pre-1915	185	147	68
2-4 Units			
Post-1944	187	3	12
1915 to 1944	293	62	42
Pre-1915	283	188	67
5+ Units			
Post-1944	96	8	13
1915 to 1944	20	5	13
Pre-1915	16	20	13
Total	1,620	592	515

SOURCE: HASE surveys of landlords at baseline, 1973 in Brown County, 1974 in St. Joseph County.

Appendix B

CONVERTING BROWN COUNTY DATA TO 1974 DOLLARS

The baseline survey of landlords obtained 1973 data for Brown County and 1974 data for St. Joseph County. To make the baseline data comparable for this analysis, we converted Brown County rent and expense data to 1974 dollars, using the inflation rates in the center column of Table B.1. As the table shows, inflation did not affect the components of housing cost uniformly in Brown County. For example, property taxes increased less than one percent, while utility costs increased more than 10 percent.

To transform capital value to 1974 dollars, we used an inflation rate of 8.2 percent. That rate is the 9.5 percent increase in the consumer price index for Milwaukee, Wisconsin* (mid-1973 to mid-1974), less 1.3 percent for estimated depreciation in capital value due to aging.

We updated mortgage debt by adding new debt during 1973 to the average mortgage debt in that year and subtracting amortization payments during the year. We estimated 1974 equity value by subtracting 1974 mortgage debt from 1974 capital value.

---

*Milwaukee is the metropolitan area closest to Brown County that is surveyed for the consumer price index.

Table B.1

CHANGE IN THE COMPONENTS OF GROSS RENT OWING TO PRICE INFLATION: BROWN COUNTY, 1973-74

Component	Fraction of Gross Rent	Inflation Rate (%) 1973-74	Contribution to Percent Change in Gross Rent 1973-74
Rent Loss			
Loss of contract rent	.042	5.8[a]	.24
Unconsumed utilities	.005	10.5[b]	.05
Services			
Utilities	.196	10.5[b]	2.06
Management	.089	8.8[c]	.78
Annual Capital Costs			
Property Tax	.171	0.7[d]	.12
Maintenance and replacement	.122	10.8[e]	1.32
Insurance	.022	7.6[f]	.17
Current return	.353	4.7[g]	1.67
Gross rent	1.000	6.4[h]	6.40

SOURCE: HASE surveys of landlords at baseline and wave 2 in Brown County; and Charles W. Noland, *Indexing the Cost of Producing Housing Services: Site I, 1973-74*, The Rand Corporation, WN-9735-HUD, April 1977 (forthcoming as N-1117-HUD).

[a] Increase in contract rent per unit for properties with no physical changes, baseline to wave 2.

[b] Table 11, WN-9735.

[c] Table 8, WN-9735.

[d] Increase in property tax per unit for properties with no physical changes, baseline to wave 2.

[e] Tables 18, 19, and 24, WN-9735.

[f] Change in value of improvements, 7.9 percent, plus change in insurance rate, -0.3 percent. Change in value is estimated by the 9.5 percent change in the consumer price index for Milwaukee, Wisconsin, less 1.6 percent depreciation. Change in insurance rate from Table 17, WN-9735.

[g] Computed as a residual.

[h] Increase in gross rent per unit for properties with no physical changes, baseline to wave 2.

Appendix C

SECOND-ORDER CONDITION FOR PROFIT MAXIMIZATION

For the first-order condition $\pi_i'(R_i) = 0$ to indicate an individual landlord's profit-maximizing rent, the second-order condition $\pi_i''(R_i) < 0$ must be satisfied. This appendix shows that the following reasonable assumptions about housing demand are sufficient for the second-order condition to be met: (a) the price elasticity of aggregate demand increases with price ($S'(R) > 0$) and (b) the additional price elasticity of demand for an individual landlord's housing services increases with that landlord's price ($T'(R_i) > 0$).

DERIVATION OF THE SUFFICIENT CONDITIONS

From Eq. (2.2) and the definition of $S_i(R_i)$ in Eq. (2.4), we know that

$$\pi_i'(R_i) = \frac{D_i(R_i)[R_i - c]}{R_i}\left[\frac{1}{1 - c/R_i} - S_i(R_i)\right] . \qquad \text{(C.1)}$$

Substituting from Eq. (2.9), we can write

$$\pi_i'(R_i) = \frac{D_i(R_i)[R_i - c]}{R_i}\left[\frac{1}{1 - c/R_i} - \frac{H_i D(R_i) S(R_i)}{H D_i(R_i)} - T(R_i)\right] . \qquad \text{(C.2)}$$

Then, because

$$\frac{d}{dR_i}\left(\frac{1}{1 - c/R_i}\right) = \frac{-c/R_i^2}{[1 - c/R_i]^2} , \qquad \text{(C.3)}$$

and further because, using Eq. (2.9) in the second and third steps,

$$\frac{d}{dR_i}\left(\frac{D(R_i)}{D_i(R_i)}\right) = \frac{D'(R_i)D_i(R_i) - D(R_i)D_i'(R_i)}{[D_i(R_i)]^2}$$

$$= \frac{D(R_i)}{D_i(R_i)R_i}\left[-S(R_i) + S_i(R_i)\right] \qquad (C.4)$$

$$= \frac{D(R_i)T(R_i)}{D_i(R_i)R_i},$$

we find that under the condition $\pi_i'(R_i) = 0$, the condition $\pi_i''(R_i) < 0$ is satisfied if and only if

$$\frac{-c/R_i^2}{[1 - c/R_i]^2} - \frac{H_iD(R_i)T(R_i)S(R_i)}{HD_i(R_i)R_i} - \frac{H_iD(R_i)S'(R_i)}{HD_i(R_i)} - T'(R_i) < 0. \qquad (C.5)$$

The first two terms in Eq. (C.5) are always negative, and the third and fourth terms will be negative if $S'(R_i) > 0$ and $T'(R_i) > 0$.

INTERPRETATION OF THE SUFFICIENT CONDITIONS

A broad range of aggregate demand curves have price elasticities of demand that increase with price ($S'(R) > 0$): concave demand curves (cupped toward the origin), linear demand curves, and convex demand curves that are less convex than a rectangular hyperbola. For the geometric interpretation, first differentiate $S(R) = -D'(R)R/D(R)$:

$$S'(R) = \frac{-D''(R)R}{D(R)} - \frac{D'(R)}{D(R)} + R\left[\frac{D'(R)}{D(R)}\right]^2. \qquad (C.6)$$

Then, holding price and the level and slope of the demand curve fixed (R, $D(R)$, and $D'(R)$ fixed), the derivative of the price elasticity of demand with respect to price $S'(R)$ decreases as the convexity of the demand curve increases (as $D''(R)$ increases). The boundary case of $S'(R) = 0$ occurs when the demand curve is a rectangular hyperbola ($D(R) = NR^{-S}$), which has a constant price elasticity of demand.

The geometric interpretation of the condition $S'(R) > 0$ assures us that many demand curves satisfy it. The assumption that actual housing demand curves satisfy the condition is therefore plausible. Appendix D illustrates the specification of the additional demand function $M(R_i)$ so as to satisfy the condition $T'(R_i) > 0$ and show that the second condition is also plausible.

Appendix D

ILLUSTRATIVE DEMAND SPECIFICATION

The specification in this appendix serves two purposes. (a) It illustrates the general argument in Sec. II and Appendix C, demonstrating that there is no pervasive logical flaw, and (b) it points to empirical work that will estimate demand functions from observed housing market behavior.

AGGREGATE DEMAND

A simple yet flexible aggregate demand curve results from multiplying a power function by an exponential function, as in

$$D = R^{-\alpha}e^{-\beta R}; \qquad \beta < 0. \tag{D.1}$$

The price elasticity of demand is a linear function of price, $S(R) = \alpha + \beta R$, and the constraint in the definition ensures that the condition $S'(R) = \beta > 0$ is always satisfied.

ADDITIONAL DEMAND

As discussed in Sec. II, the additional demand for an individual landlord's housing services must be a decreasing function of that landlord's rent, and further, must be zero when his rent equals the average rent. The logarithm of the ratio of average rent to individual rent has those characteristics. To complete our example, we assume that the additional demand caused by a given amount of price-cutting is directly proportional to aggregate demand and inversely proportional to the average occupancy rate:

$$M = \gamma \; ln\left(\frac{R}{R_i}\right)\frac{D(R)}{F} . \tag{D.2}$$

Since by definition $F = D(R)/H$, Eq. (D.2) can be rewritten as

$$M = \gamma H \; ln\left(\frac{R}{R_i}\right). \tag{D.3}$$

Differentiating Eq. (D.3) with respect to R_i, and recognizing that the individual landlord's contribution to average rent is proportional to his market share so that $R'(R_i) = H_i/H$, yields

$$M'(R_i) = -\gamma\left[\frac{H}{R_i} - \frac{H_i}{R}\right]. \tag{D.4}$$

Constructing the additional price elasticity of demand yields

$$T = \frac{-M'(R_i)R_i}{D_i(R_i)} = \frac{\gamma H}{D_i(R_i)}\left[1 - \frac{H_iR_i}{HR}\right]. \tag{D.5}$$

Then, differentiating T with respect to R_i, again using $R'(R_i) = H_i/H$, gives

$$T'(R_i) = \frac{\gamma H}{[D_i(R_i)]^2}\left[-\frac{H_i}{H}\left(\frac{R - R_iH_i/H}{R^2}\right)D_i(R_i) - \left(1 - \frac{H_iR_i}{HR}\right)D_i'(R_i)\right]. \tag{D.6}$$

Since $S_i(R_i) = -D_i'(R_i)R_i/D(R)$, we obtain

$$T'(R_i) = \frac{\gamma H}{D_i(R_i)R}\left[1 - \frac{H_iR_i}{HR}\right]\left[\frac{RS_i(R_i)}{R_i} - \frac{H_i}{H}\right]. \tag{D.7}$$

In shortrun equilibrium, $R_i = R$; from Eq. (2.4) $S_i(R_i) > 1.0$. Therefore, since $H_i/H < 1.0$, the condition $T'(R_i) > 0$ is always satisfied.

CONCLUSION

In shortrun equilibrium, the additional price elasticity of demand has a particularly simple form in our example. Rewriting Eq. (D.5) using $R_i = R$, $D_i(R_i) = [H_i/H]D(R_i)$, and recognizing that $F = D(R)/H$, yields

$$T = \frac{A}{F}\;; \qquad A = \gamma\left[\frac{H}{H_i} - 1\right]. \tag{D.8}$$

Substituting $T'(F) = -A/F^2$ (note that it is always negative) and our earlier specification that $S'(R) = \beta$ into Eq. (2.13) gives the equilibrium price elasticity of the occupancy rate:

$$Z = \frac{F}{A}\left[\beta R + \frac{c/R}{[1 - c/R]^2}\right]. \tag{D.9}$$

Using the estimates $Z = 3.4$, $S = 0.5$, $c = 320$, $R = 1{,}760$, $T = 0.7$ (from Eq. (2.11)), and $F = 0.94$, Eqs. (D.1), (D.8), and (D.9) are solved to yield parameters of $\gamma = -1{,}613$, $\beta = 0.0012$, and $A = 0.658$ for our illustrative demand specification.

We conclude that given the demand specification in Eqs. (D.1) and (D.2) and the parameter estimates in this report, the price elasticity of aggregate demand S equals $-1{,}613 + 0.0012\ R$ (which is 0.5 when rent is 1,760 and increases when rent increases), and the additional price elasticity of demand for an individual landlord's housing services T equals $0.658/F$ (which is 0.7 when the occupancy rate is 0.94 and decreases as the occupancy rate increases).

REFERENCES

Barnett, C. Lance, and Ira S. Lowry, *How Housing Allowances Affect Housing Prices*, The Rand Corporation, R-2452-HUD, September 1979.

Blank, David M., and Louis Winnick, "The Structure of the Housing Market," *Quarterly Journal of Economics*, Vol. 67, No. 2 (May 1953), pp. 181-208.

de Leeuw, Frank, and Nkanta F. Ekanem, "Time Lags in the Rental Housing Market," *Urban Studies*, Vol. 10 (1973), pp. 39-68.

Fourth Annual Report of the Housing Assistance Supply Experiment, The Rand Corporation, R-2302-HUD, May 1978.

Mayo, Stephen K., *Theory and Estimation in the Economics of Housing Demand*, paper presented at the American Economic Association, Chicago, Illinois, 29-31 August 1978.

Mulford, John E., *Income Elasticity of Housing Demand*, The Rand Corporation, R-2449-HUD, July 1979.

Muth, Richard F., *Cities and Housing*, University of Chicago Press, Chicago, Illinois, 1969.

Noland, Charles W., *Indexing the Cost of Producing Housing Services: Site I, 1973-74*, The Rand Corporation, WN-9735-HUD, April 1977 (forthcoming as N-1117-HUD).

Rapkin, Chester, Louis Winnick, and David M. Blank, *Housing Market Analysis: A Study of Theory and Methods*, The Institute for Urban Land Use and Housing Studies, Columbia University, 1953.

RAND/R-2453-HUD