SpAM

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Measuring Spatial Mismatch Between Homelessness and Homeless Resources With a Theil Index and Statistical Inference

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The views expressed in this article are those of the author and do not represent the official positions or policies of the Office of Policy Development and Research, the U.S. Department of Housing and Urban Development, or the U.S. government.

Abstract

In this article, I employ a Theil (1972) index to measure the spatial mismatch of beds available to shelter the homeless and homeless populations across Continuum of Care regions. I demonstrate a method for statistical inference using the Theil index based on asymptotic results, focusing mainly on testing for across-state differences. Estimates reveal large differences across states in the spatial mismatch between homeless resources and homeless populations. Simulations indicated that state inferences are better for states that have a relatively larger estimated spatial mismatch and relatively larger total count of beds available to shelter the homeless.

The purpose of this article is to demonstrate a method for measuring the spatial mismatch between homelessness and resources for the homeless. Widely used to measure economic inequality (for example, see Conceição and Galbraith, 2000), Theil indices (Theil, 1972, 1967) have also been used extensively to measure racial segregation (for example, see Wong, 2003) and other inequalities such as disparities in health measures (Borell and Talih, 2011). Wilson (2011a, 2011b) previously introduced *Cityscape* readers to a Theil index.

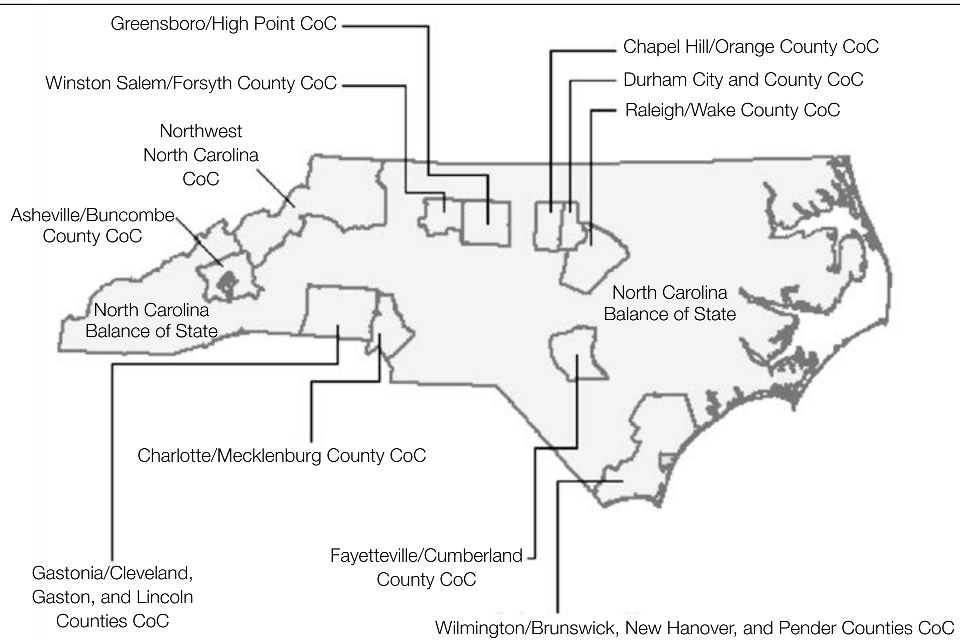
The superiority of Theil indices compared with other inequality measures, such as the Gini coefficient, is well established based on their mathematical properties (for example, see Reardon and Firebaugh, 2002), yet no consensus exists regarding their use for statistical inference (for one approach, see Cowell and Victoria-Feser, 2003).

This article demonstrates statistical inference with a Theil index based on asymptotic results using a data example to measure the spatial mismatch between resources available to shelter the homeless and homeless populations.

The geographic units on which the spatial mismatch measure is based are 421 Continuum of Care (CoC) regions in the 50 states and Washington, D.C. CoCs are a consortium of providers within defined areas (within states) that provide a broad range of housing and services to homeless populations. Maps of CoC regions are available on the U.S. Department of Housing and Urban Development's (HUD's) CoC Maps website (HUD, 2013a). For example, exhibit 1 is a map of the 12 CoCs in North Carolina for 2012.

Exhibit 1

North Carolina CoC Regions



CoC = Continuum of Care.

Source: http://www.hudhre.info/index.cfm?do=viewCocMaps&stateAbbreviation=NC&yr=2012#tab

HUD's CoC Program is the largest single source of federal funding used to reduce U.S. homelessness.[1] HUD's CoC Program provides assistance to local CoCs through homeless assistance programs. In 2012, HUD awarded $1.7 billion to local CoCs in the United States, Puerto Rico, and other U.S. territories. The corresponding estimated homeless population in 2012 was 633,782, of which 390,155 were estimated to be sheltered. I use bed counts by CoC as a measure of available resources. In 2012, an estimated 476,119 beds were available to shelter the homeless in the United States, Puerto Rico, and other U.S. territories.

The main focus of this analysis is to test for differences in spatial mismatch of homeless resources and homeless populations across states. Estimates reveal large spatial mismatch differences across states. Simulations indicate inferences based on asymptotic approximations are more accurate for states with (1) greater estimated spatial mismatch, (2) greater total counts of beds available to shelter the homeless, and (3) greater bed counts per CoC. I also demonstrate how the technique can be adapted to test for significant differences within a single state.

Homelessness and Homeless Resources

I measure spatial mismatch by examining total beds and total homeless populations by CoC. My measure could be extended to model the spatial mismatch of categories of homeless resources with categories of homeless populations.[2]

Data Sources

Data on homeless population estimates by CoC for this analysis are from HUD's 2012 CoC Homeless Populations and Subpopulations Reports (HUD, 2013c). These reports use Point-in-Time (PIT) data provided to HUD by CoCs when they apply to HUD for housing assistance. PIT data provide count estimates of homeless populations (sheltered and unsheltered) and homeless subpopulations (for example, number of individuals, number of people in families, number of chronically homeless, and number of veterans) on a single night within the last 10 days in January.

Data on the number of beds available to shelter the homeless population by CoC for this analysis are from HUD's 2012 CoC Housing Inventory Count (HIC) Reports (HUD, 2013d). HUD's HIC Reports are based on data provided to HUD by CoCs. HIC data collection is also conducted on a single night within the last 10 days in January. The HIC data contain information on the number of available beds and housing units dedicated to homeless populations. The HIC data also capture information on housing categories (for example, emergency shelter, transitional housing, number of beds for households without children, number of beds for households with children, and number of permanent beds for the chronically homeless), along with a measure of unmet housing needs.

[1] See HUD (2013b) for information about CoCs and HUD's CoC Program. See Burt et al. (2002) for an evaluation of HUD's CoC Program.

[2] Performing a cluster analysis using New York City and Philadelphia administrative data, Kuhn and Culhane (1998) categorized homeless populations into three groups: transitional, episodic, and chronic. They found large demographic differences and differences in rates of mental health, substance abuse, or medical problems across the groups. They also found large differences across groups in how homeless resources were used. The chronically homeless accounted for 10 percent of shelter users but consumed one-half of total shelter days.

Data Limitations

Although HUD provides guidance for collecting PIT and HIC data (HUD, 2013e), compliance among CoCs may vary. Therefore, the reliability and consistency of these data may also vary among CoCs.

Another limitation of the data is that no national source of data on the distribution of homeless populations and homeless resources within CoCs exists. Six low-population states (Delaware, Montana, North Dakota, Rhode Island, South Dakota, and Wyoming) and Washington, D.C., comprise a single CoC region. As such, it is not possible to accurately measure the spatial mismatch of homeless resources and homeless populations within these jurisdictions.

Further, in rural areas, large distances can exist between homeless populations and homeless resources.[3] Hence, spatial mismatch may be greater within rural CoCs, such as Wyoming, compared with more urban CoCs, such as Washington, D.C.

Spatial mismatch measurements in the 44 states that have multiple CoCs are limited to measurements made across CoCs. In general, measurement should be more accurate as total homeless resources or total homeless populations decrease relative to the number of CoCs within a state. Among the 44 states that had multiple CoCs in 2012, the ratio of the estimated homeless population to the number of CoCs varied from 443 homeless people in Virginia, which had 19 CoCs, to 5,589 homeless people in Colorado, which had 3 CoCs.

Spatial Mismatch

John Kain's pioneering research (1968, 1964) focused heavily on the spatial mismatch of job seekers and available jobs. (For a recent example, see Li, Campbell, and Fernandez, 2013. For reviews, see Kain, 2004; Ihlanfeldt and Sjoquist, 1998; and Kain, 1992.) Kain's (1968) spatial mismatch hypothesis posited that—

> … the suburbanization of jobs and involuntary housing market segregation have acted together to create a surplus of workers relative to the number of available jobs in submetropolitan areas where blacks are concentrated. (Ihlanfeldt and Sjoquist, 1998: 849)

Studies of labor market spatial mismatch have used various measures. Andersson et al.'s (2011) analysis was based on counts of accessible jobs and job searchers by census tract and travel time. Li, Campbell, and Fernandez (2013) used racial and skill-based dissimilarity indices computed by census tract.

Theil Indices

Theil (1972, 1967) proposed various related inequality indices (for a discussion, see Frenken, 2007). Reardon and Firebaugh (2002) compared numerous measures of multigroup disproportionality (including a Theil [1972] index, the dissimilarity index, and the Gini index) and found a Theil (1972) index was "the most conceptually and mathematically satisfactory" (Reardon and Firebaugh, 2002: 33).

[3] For a discussion of rural homelessness, see Robertson et al. (2007).

In a spatial context, a Theil index can be used to measure disproportionality across geographic units. Novotný (2007) examined the use of a Theil index to measure income inequality in spatially defined subgroups.

My index is based on Theil's (1972: 59) index, commonly referred to as Theil's T index, or simply "the Theil index" (Conceição and Ferreira, 2000). Let π_i represent the share of beds available to shelter the homeless in CoC i, w_i represent the share of the homeless population in CoC i, w_{min} represent the minimum of the w_is, and let J represent the total number of CoCs. I will define a normalized Theil index Tn by the following equation:

$$Tn = \frac{\sum_1^J \pi_i \log\left(\frac{\pi_i}{w_i}\right)}{\log\left(\frac{1}{w_{min}}\right)}, \qquad (1)$$

where log denotes the natural logarithm. The numerator of equation (1) is Theil's T, which is the weighted summation of the logarithms of the bed share to homeless share ratios; weights are the bed shares. The denominator is a normalizing constant to constrain the index's maximum value to 1.

Thus, if the bed share equals the homeless share in each CoC, then the index equals 0 and indicates no spatial mismatch. That is, the geographic location of beds and the homeless population are matched perfectly. If all beds are in the CoC with the smallest share of the homeless population, then the index equals 1. In other words, maximum spatial mismatch exists between resources for combating homelessness and homeless populations.

Statistical Inference With a Theil Index

Exact inference with a Theil index requires derivation of its probability distribution, which presents statistical challenges. Cowell and Victoria-Feser (2003) presented distribution-free (nonparametric) approaches. Martinez-Camblor (2007) suggested approximate inference, treating a Theil income inequality index as asymptotically normal. Biewen and Jenkins (2006) derived variance estimators for Theil-type indices with complex survey data.

Results for Theil indices based on random variables with continuous distributions, such as income (Martinez-Camblor, 2007) or large population counts with distributions that can safely be treated as continuous, may not hold for Theil indices based on discrete random variables such as bed counts.

The following method is used for approximate inference based on asymptotic results assuming the distribution of the number of beds among CoCs follows a multinomial distribution. Let $X = (X_1, X_2, \ldots, X_J)$ represent a vector of cell counts following a multinomial (n, π) distribution, where $\pi = (\pi_1, \pi_2, \ldots, \pi_J)$ is a vector of cell probability parameters summing to 1. Also let $p = (p_1, p_2, \ldots, p_J)$ represent a vector of sample cell proportions where $p_i = X_i / \sum_1^J X_i$. Based on the multivariate central limit theorem (Rao, 1973: 128), Agresti (2013: 590) proved the asymptotic normality of p.

Let $g(p)$ be a differentiable function of the sample cell proportions, and let $\varphi_i = \partial g / \partial \pi_i$ denote $\partial g / \partial p_i$ (the partial derivative of g with respect to p_i) evaluated at $p = \pi$. Using the delta method (for example, see Casella and Berger, 2002: 243–245), Agresti (2013: 590) derived the asymptotic variance of $g(p)$ to be

$$\sum_1^J \pi_i \varphi_i^2 - \left[\sum_1^J \pi_i \varphi_i\right]^2. \qquad (2)$$

From (1),

$$\partial Tn/\partial \pi_i = \frac{log\,(1 + \frac{\pi_i}{w_i})}{log\,(\frac{1}{w_{min}})}. \tag{3}$$

Substituting (3) for φ_i in (2) (and defining cells as CoCs) results in an asymptotic variance formula for Tn:

$$V = \frac{\sum_1^J \pi_i\,[log\,(1 + \frac{\pi_i}{w_i})]^2 - [\sum_1^J \pi_i\,log\,(1 + \frac{\pi_i}{w_i})]^2}{[log\,(\frac{1}{w_{min}})]^2}. \tag{4}$$

The model treats the homeless population shares ($w_i s$) as known constants and the bed shares ($\pi_i s$) as unknown parameters. We can obtain a Theil index estimate $\widehat{Tn}$ by substituting the sample bed shares for the $\pi_i s$ into (1). We can obtain an asymptotic variance estimate $\widehat{V}$ by substituting the sample bed shares for the $\pi_i s$ into (4).

An approximate 95-percent confidence interval for Tn is

$$\widehat{Tn} \pm 1.96\sqrt{\frac{\widehat{V}}{n}}, \tag{5}$$

where n is the total sample bed count. We set any estimated lower confidence limits less than 0 to 0.

We can test whether the difference between two independent estimates of Tn are statistically significant at the .05 level by checking whether their 95-percent confidence intervals overlap. We can perform a more general Z test using the following formula:

$$Z = \frac{(\widehat{Tn}_1 - \widehat{Tn}_2) - D_0}{\sqrt{\frac{\widehat{V}_1}{n_1} + \frac{\widehat{V}_2}{n_2}}}, \tag{6}$$

where 1 and 2 denote the independent samples and D_0 denotes the difference under the null hypothesis.

Estimates

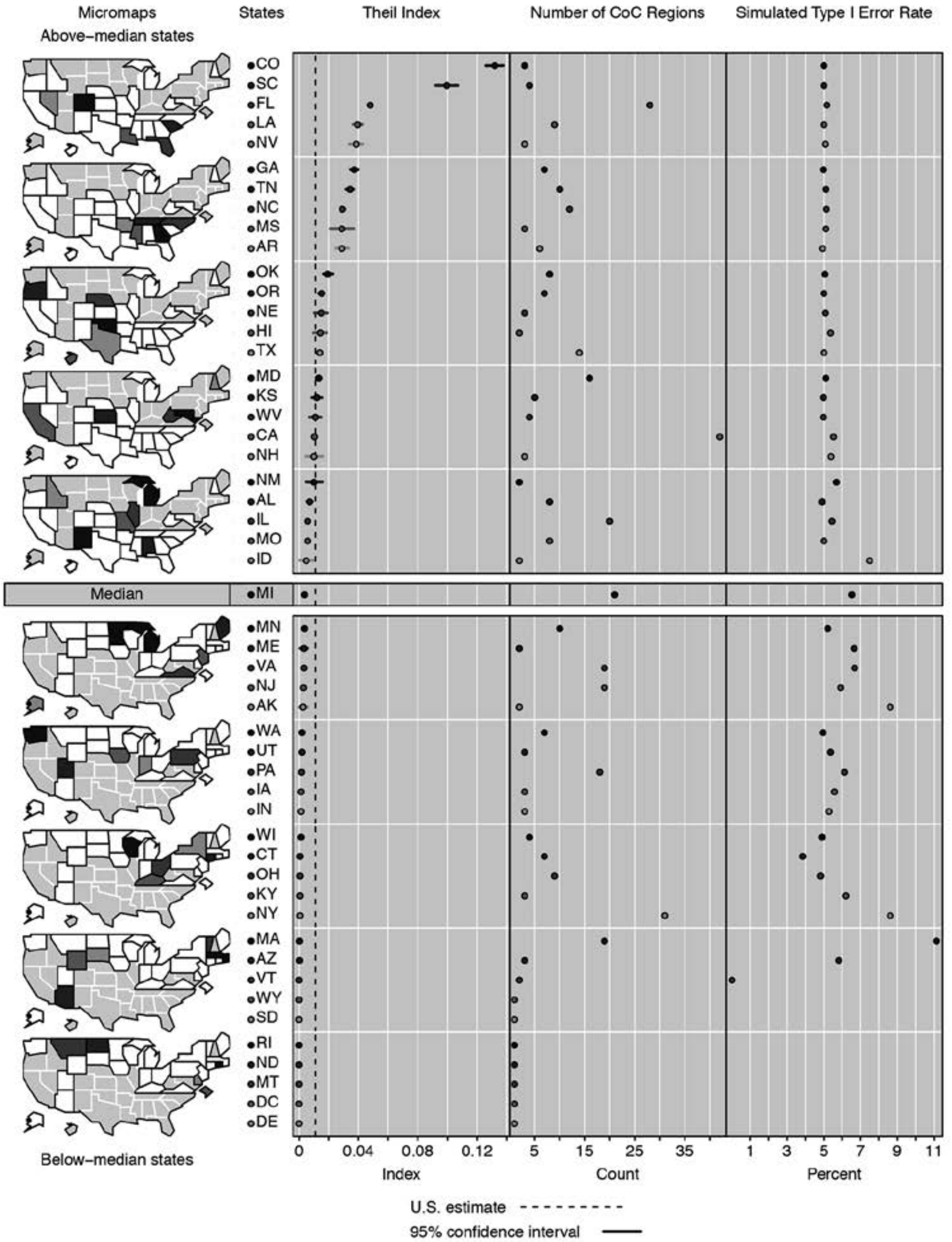
Measured across 421 CoCs in the 50 states and Washington, D.C., the national estimated Tn equals .0108 with a 95-percent confidence interval of (.0107, .0110). The 95-percent confidence interval excludes 0; thus, we can reject the null hypothesis that beds and homeless populations are distributed evenly across all CoCs. The fact that the lower confidence limit is close to 0 suggests that the total amount of mismatch across CoCs in the United States is quite modest.

Exhibit 2 reports a linked micromap (for example, see Carr and Pickle, 2010) with Theil indices computed by state. Mast (2013) previously introduced *Cityscape* readers to linked micromaps. The first column of data in exhibit 2 contains state Theil index estimates reported in descending order, with 95-percent confidence intervals. The second column of data in exhibit 2 reports the number of CoCs per state.

The seven observations with only one CoC have an estimated Theil index equal to 0 and an estimated variance of 0. This variance estimate does not imply that no spatial mismatch exists within these six states and Washington, D.C. Rather the 0 estimates reflect a lack of data on the distribution of beds and homeless counts within CoCs.

Exhibit 2

State Theil Index Estimates



Source: HUD Point-in-Time and Housing Inventory Count data, 2012

The first quartile estimate is .0006 in Ohio with a 95-percent confidence interval of (.0003, .0009); Ohio has 9 CoCs. The median estimate is .0035 in Michigan with a 95-percent confidence interval of (.0029, .0041); Michigan has 21 CoCs. The third quartile estimate of .0147 is in Nebraska with a 95-percent confidence interval of (.0103, .0191); Nebraska has three CoCs. The maximum estimated mismatch is in Colorado, where the estimated index is .1320 with a 95-percent confidence interval of (.1261, .1378); Colorado has 3 CoCs.

Maryland has an estimated Theil index of .0131 and a 95-percent confidence interval of (.0115, .0146). Hawaii's estimated Theil index is .0141, with a 95-percent confidence interval of (.0097, .0185), which overlaps with Maryland's. Thus, we cannot reject the null hypothesis that Maryland and Hawaii have the same Theil index at the .05 significance level. The Z test statistic under a null hypothesis of no difference in the Theil indices in Maryland and Hawaii is -.42; the p-value for a two-tailed test is .67.

Illinois has an estimated Theil index of .0059, with a 95-percent confidence interval of (.0050, .0067). Because Maryland's and Illinois' confidence intervals do not overlap, we can reject the null hypothesis that Maryland and Illinois have equal Theil indices at the .05 significance level. The Z test statistic under a null hypothesis of no difference is 8.05, with a two-sided p-value less than .0001.

Simulations

I conducted simulations to test the reliability of state asymptotic inferences. For each state with multiple CoCs, I generated 1 million random samples from a multinomial (n, p) distribution, where n is the state's sample bed count and $p = (p_1, p_2, \ldots, p_J)$ is a vector of the state's sample bed shares across the J CoCs. For each iteration of the simulation, I computed a 95-percent confidence interval for the state's Theil index.

The percentage of a state's simulated confidence intervals that excludes the state's estimated Theil index approximates a type I error rate (referred to as the α level, or size of the test). A type I error is an incorrect rejection of a true null hypothesis. For my simulation, I treat the state's estimated Theil index as the true value under the null hypothesis. If the asymptotic inferences are accurate, the type I error rate associated with 95-percent confidence intervals should be 5 percent.

The simulated type I error rates are reported in the third column of data in exhibit 2. These error rates are closer to 5 percent for estimated Theil indices farther from 0. Among the 44 states with multiple CoCs, 18 have estimated Theil indices below Michigan's (the median) estimate. Of these 18 states, 9 have simulated type I error rates between 4 and 6 percent. Of the 25 states with estimated Theil indices above Michigan's, 24 have simulated type I error rates between 4 and 6 percent. The exception is Idaho, with an estimated Theil index of .0048, ranked 25th, and a simulated α level of 7.5 percent.

The inferences are based on an assumption of asymptotic normality, which is more likely to hold with higher bed counts. Among the 44 states with multiple CoCs, 19 have bed counts below Louisiana's count of 5,256 (the median). Of these 19 states, 13 have simulated type I error rates between 4 and 6 percent. Among the 44 states with multiple CoCs, 24 have state bed counts above Louisiana's. Of these 24 states, 19 have simulated type I error rates between 4 and 6 percent.

Among the 44 states with multiple CoCs, 23 have mean bed counts per CoC below Utah's mean of 1,158.3 beds (the median). Of these 23 states, 15 have simulated type I error rates between 4 and 6 percent. Among the 44 states with multiple CoCs, 20 have mean bed counts per CoC above Utah's. Of these 20 states, 17 have simulated type I error rates between 4 and 6 percent.

Within-State Inference

For states with at least four CoCs, we can test for differences between subgroups of CoCs within states. For example, consider North Carolina, which has a state Theil index estimate of .0292 with a confidence interval of (.0276, .0308) and 12 CoCs (see exhibit 1). Let group one consist of the 3 CoCs in the Research Triangle area of the state: the city of Chapel Hill and Orange County; the city of Durham and Durham County; and the city of Raleigh and Wake County. Let group two consist of the remaining 9 CoCs. The estimated Theil index for group one is .0016, with a 95-percent confidence interval of (.0001, .0032). The estimate for group two is .0514, with a 95-percent confidence interval of (.0485, .0543).

Because the two groups' confidence intervals do not overlap, we can reject the null hypothesis of equal Theil indices at the .05 significance level. The Z test statistic under a null hypothesis of no difference is -29.83, with a two-sided p-value less than .0001. Very strong evidence suggests that beds are distributed more proportionately to the homeless population in the 3 CoCs in the Research Triangle area of the state compared with the remaining 9 CoCs.

Conclusion

In this study, I used a Theil index to measure the spatial mismatch of beds available to shelter homeless (homeless resources) and homeless populations across CoC regions. I demonstrated statistical inference using the Theil index based on asymptotic results. I focused mainly on testing for differences in spatial mismatch across states. I find large differences across states in the spatial mismatch between homeless resources and homeless populations.

I also performed simulations to assess the reliability of the asymptotic inferences. Simulations revealed that state inferences are better for states that have a relatively larger estimated spatial mismatch and relatively larger total count of beds available to shelter the homeless.

I also demonstrated how this asymptotic inference method can be adapted to test for significant differences in spatial mismatch within states.

Policy Implications

Using the technique described in this article can further the efficient and effective allocation of scarce resources for serving homeless populations. This type of analysis can help inform decision-makers who can ensure that beds are available in the areas that have the greatest need. The practical application of this method is that it can suggest more optimal alternatives for deploying beds and other resources for the homeless within and among jurisdictions.

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